Seminar: The Stochastic Filtering Problem

Summer Term 2019

http://belak.ch/the-filtering-problem-2019/

Technische Universität Berlin

The Filtering Problem

Signal Process:

$$X = \{X_t\}_{t \geq 0}$$

Observation Process:

$$Y_t = \int_0^t X_s \, \mathrm{d}s + W_t, \qquad t \geq 0.$$

Task: Try to estimate X. You can only observe Y.

Easy Answer: Least-squares optimal estimate is $\hat{X} = \{\hat{X}_t\}_{t \geq 0}$ with

$$\hat{X}_t = \mathbb{E}\big[X_t \big| \sigma(Y_s : s \leq t)\big], \qquad t \geq 0.$$

Topics

Basics:

(1) The Filtering Problem in Discrete Time

(2) The Filtering Problem in Continuous Time and the Kalman Filter

(3) Hidden Markov Models and the Wonham Filter

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Further Topics:

(4) Parameter Estimation with the EM Algorithm

(5) Parameter Uncertainty in the Kalman Filter

(6) Filter-Based Stochastic Volatility

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Further Topics:

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Applications:

(7) Portfolio Optimization in Hidden Markov Models
(8) Portfolio Optimization with Partial Information with Gaussian Drift
(9) Portfolio Optimization with Partial Information and Expert Opinions
(10) Randomization in Control Problems with Partial Information

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- Questions?

(1) Filtering in Discrete Time

Problem: Stochastic filtering in discrete time, i.e.

$$Y_{k+1} = c(X_k, w_{k+1}), \qquad k \in \mathbb{N},$$

where X is a finite-state Markov chain and w iid noise. The aim is to obtain the best estimate for X given the observation of Y and estimate various properties of X.

In this talk, the fundamental ideas of filtering are introduced in a tractable setting.

Prerequisites: Discrete-time stochastic processes, Markov chains.

(2) Filtering in Continuous Time; Kalman Filter

Problem: Stochastic filtering in continuous time, i.e.

$$Y_t = \int_0^t c(s, X_s, Y_s)\,\mathrm{d}s + \int_0^t \alpha(s, Y_s)\mathrm{d}W_s, \qquad t \geq 0,$$

for a general X. The aim is to obtain the best estimate for X given the observation of Y and estimate various properties of X. Moreover, computation of the filter for the special case when X is a Gaussian process of the form

$$X_t = X_0 + \int_0^t aX_s\mathrm{d}s + bB_t, \qquad t \geq 0.$$

In this talk, the two fundamental approaches to filtering in continuous time are introduced: The innovations approach and the reference probability approach. Moreover, the famous Kalman-Bucy filter is introduced.

Prerequisites: Stochastic integration and differential equations, Itō's formula, Girsanov's theorem, martingale representation.

Helpful: Semimartingales, quadratic variation.

(3) Hidden Markov Models; Wonham Filter

Problem: Stochastic filtering in continuous time, i.e.

$$Y_t = \int_0^t c(X_s)\,\mathrm{d}s + W_t, \qquad t \geq 0,$$

where X is a finite-state Markov chain in continuous time. This leads to the so-called Wonham filter.

In this talk, an introduction to continuous time Markov chains is given. Moreover, the important Wonham filter is introduced.

Prerequisites: Stochastic integration and differential equations, Itō's formula, Girsanov's theorem, martingale representation.

Helpful: Semimartingales, quadratic variation, no fear of jump processes.

(4) EM Algorithm

Problem: In filtering problems, the signal process is typically parametric, e.g.

$$X_t = X_0 + \int_0^t a X_s \mathrm{d}s + b B_t, \qquad t \geq 0.$$

How can the parameters (here: a and b) of an unobservable process be estimated?

In this talk, the expectation maximization (EM) algorithm for parameter estimation is introduced and applied to filtering problems.

Prerequisites: Stochastic integration and differential equations, Itō's formula, Girsanov's theorem, martingale representation.

(5) Parameter Uncertainty

Problem: In filtering problems, the signal process is typically parametric, e.g.

$$X_t = X_0 + \int_0^t aX_s \mathrm{d}s + bB_t, \qquad t \geq 0.$$

When estimating the parameters (here: a and b), there is statistical uncertainty. Can we define a filter which takes this statistical uncertainty into account?

In this talk, a robust filter taking into account statistical uncertainty on the model parameters of the signal is introduced in the special case of the Kalman-Bucy filter.

Prerequisites: Stochastic optimal control, viscosity solutions.

(6) Filter-Based Volatility

Problem: In continuous time, the filtering problem for

$$Y_t = \int_0^t aX_s \, \mathrm{d}s + \int_0^t bX_s \mathrm{d}W_s, \qquad t \geq 0,$$

is trivial: X can be recovered from Y by computing the quadratic variation

$$\langle Y \rangle_t = \int_0^t b^2 X_s^2 \mathrm{d}s.$$

In discrete time, this problem does not arise, i.e. discrete and the continuous time models are inconsistent.

In this talk, the inconsistency is discussed and a stochastic volatility model

$$Y_t = \int_0^t aX_s \, \mathrm{d}s + \int_0^t b_s \mathrm{d}W_s, \qquad t \geq 0,$$

consistent with the discrete time model is introduced.

Prerequisites: Stochastic integration and differential equations, Itō's formula, Girsanov's theorem, martingale representation.

(7) Optimal Investment; Hidden Markov

Problem: Optimize utility of terminal wealth in a model with stock price

$$P_t = p + \int_0^t \mu_s P_s \, \mathrm{d}s + \int_0^t \sigma P_s \, \mathrm{d}W_s, \qquad t \geq 0,$$

with unobservable drift μ modeled as a continuous time Markov chain.

In this talk, the problem of optimal investment in a Hidden Markov model is solved.

Prerequisites: Stochastic integration and differential equations, Itō's formula, financial mathematics.

Helpful: Stochastic optimal control.

(8) Optimal Investment; Gaussian Drift

Problem: Optimize utility of terminal wealth in a model with stock price

$$P_t = p + \int_0^t \mu_s P_s \, \mathrm{d}s + \int_0^t \sigma P_s \, \mathrm{d}W_s, \qquad t \geq 0,$$

with unobservable drift Gaussian drift

$$\mu_t = \int_0^t \alpha(\beta - \mu_s)\mathrm{d}s + \gamma B_t, \qquad t \geq 0.$$

In this talk, the problem of optimal investment with unobservable drift is solved.

Prerequisites: Stochastic optimal control.

(9) Optimal Investment; Expert Opinions

Problem: Optimize utility of terminal wealth in a model with stock price

$$P_t = p + \int_0^t \mu_s P_s \, \mathrm{d}s + \int_0^t \sigma P_s \, \mathrm{d}W_s, \qquad t \geq 0,$$

with unobservable drift Gaussian drift

$$\mu_t = \int_0^t \alpha(\beta - \mu_s)\mathrm{d}s + \gamma B_t, \qquad t \geq 0.$$

The filter for the drift is updated by exogenuous expert opinions.

In this talk, the problem of optimal investment with unobservable drift with additional expert opinions is solved and the effect of various information structures discussed.

Prerequisites: Stochastic integration and differential equations, Itō's formula, financial mathematics.

(10) Randomization for Partially Observed Control Problems

Problem: Optimize a functional of the state process

$$X_t = x + \int_0^t b(X_s, \alpha_s)\,\mathrm{d}s + \int_0^t \sigma(X_s, \alpha_s)\,\mathrm{d}W_s, \qquad t \geq 0,$$

over the choice of processes α adapted to a filtration smaller than the natural filtration of W. In this case, the filter depends on the choice of α.

In this talk, this type of partially observed optimal control problem with control-dependent filter is studied.

Prerequisites: Stochastic optimal control, analysis on the Wasserstein space.

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Please choose (and order) your preferred three topics!